The Mathematics of Scheduling

Alexis Martinez & Dr. Ana Vasilic

Dept. of Mathematics and Physical Sciences, Northern New Mexico College

Abstract

Optimization problems of minimizing the cost and/or time to finish a task under given constraints are used in everyday life, from production, to managing everyday activities in work. The focus of this project is a study of different scheduling techniques from a mathematical point of view. After studying different algorithms to make optimal schedules, we would like to apply those mathematical methods to the real-world problems related to Santa Claran Travel Center operation.

Terminology

Processors are what will carry out the job. Processors could be anything from an actual person to a machine that is designed to build and create items.
Tasks are parts of the job that the processors will be carrying out.
Processor time is the amount of time it takes the processor to carry out given task.
Precedence relations tell us which tasks must be carried out before other tasks.
Finishing time for a project is the total amount of time it takes to complete all tasks.
Optimal finishing time is the least amount of time it takes for all the tasks to be completed.

Task (Time in hours)	Precedent Tasks
Admiral Order: A (2)	
Bluesky Order: B (1)	c
Core Mark Order: C (3)	
Cola-Cola Order: CC (3)	
Premier Order: P (1)	
Admiral Delivery: a(2)	A
Bluesky Delivery: b(2)	B
Core Mark Delivery: c(3)	C, a
Coca Cola Delivery: cc(3)	CC
Premier Delivery: p(1)	P, c

Table 1. Tasks, processing times, and precedence relations.

Introduction

The aim of math scheduling is to create algorithms that lead to the optimal schedules. Scheduling constraints could be represented in various ways. Using graphs, tables, or charts are the most common representations. There are also various approaches and algorithms one can use to decide on the schedule.

Decreasing-Time Algorithm is the process of creating a schedule using a list in which the tasks are placed in decreasing order of their processing times. This algorithm is not very well suitable for the constraints we have in Table 1. Even though Core Mark's order and delivery has a longer processing time, Admiral's order and delivery must be completed first. Decreasing-Time Algorithm works well with more flexible tasks.

Critical-Path Algorithm is the process of creating a schedule using a list in which the tasks are in decreasing order of critical times. A critical time for a task is the time to complete that task and the following tasks all the way to the End. In Chart 1, the critical path would be the path starting with Task A and ending at Task b with a total of 13 hours.

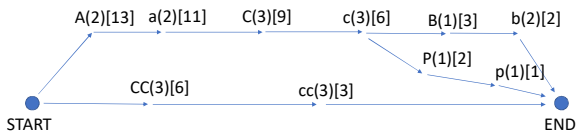

Chart 1. This timeline represents each task and it's precedence relation. The critical path is labeled by [ ].

Some Challenges

- All deliveries need to be done one day after the corresponding orders.
- There is a maximum of 5 hours for the AM shift, and 3 hours for the PM shift.
- All precedence relations need to be followed. For example, the Tasks B and P could not be started before all A,a, C, and c were done (which takes a total of 10 hours).

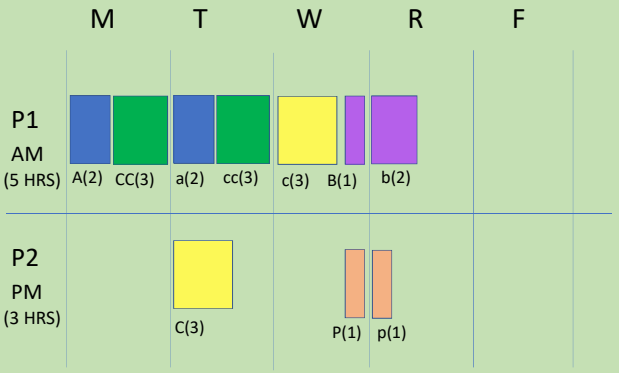

Chart 2. Tasks distributed throughout the week, following the precedence relations.

Discussion

We can see some examples of the precedent tasks in Table1 as well as in Chart 2. Tasks B/b and P/p cannot be completed until Tasks C/c have been completed. We were able to use this data to create a timeline illustration in Chart 1 to show how some tasks can be carried out by different paths.

In Chart 2 above, there is a total of two processors. The processors have been broken into an AM shift and a PM shift. The AM shift has a total of 5 hours available for orders/deliveries while the PM shift has a total of 3 hours available. Orders and deliveries for each company are distributed throughout the week. This chart follows precedence relations because Core Mark's (C/c) order and delivery have been completed before Bluesky's (B/b) and Premier's (P/p) orders and deliveries can start. Both Admiral (A/a) and Coca-Cola (CC/cc) are tasks with no precedence relations and can be completed first.

This chart allows most of the orders and deliveries to be completed at the beginning of the week. Three different orders and two deliveries have been completed in two days (Monday & Tuesday). The last two orders and three deliveries are completed the following two days (Wednesday & Thursday). This is a time effective chart because there are no orders or deliveries that need to be completed on Friday. This allows workers to complete more tasks around the store when no time is needed for completing orders or scanning in deliveries.

References

- Baker, K. R. & Trietsch, D. (2018). *Principles of Sequencing and Scheduling.* In: *Series in Operations Research and Management Science.* Wiley.
- Graham, R. L. (1978). *Combinatorial Scheduling Theory*. In: Steen L.A. (Ed.) *Mathematics Today Twelve Informal Essays*. Springer, New York, NY.
- Hunter, D. J. (2017). *Essentials of Discrete Mathematics*. Jones & Bartlett Learning, Burlington, MA.
- Tannenbaum, P. (2014). *Excursions in Modern Mathematics*. Pearson, Boston, MA.

Contact info:
alexis_n_martinez@nnmc.edu
ana.vasilic@nnmc.edu